Estimating unsaturated hydraulic conductivity from air permeability

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Abstract

Reliable functions relating unsaturated hydraulic conductivity($K(\theta)$) to air permeability (K_a) may greatly facilitate $K(\theta)$ prediction. In the current research $K(\theta)$ and K_a were measured by pressure plate outflow and variable head methods, respectively, in the range of 0 to -10 mH_2O matric potential (ψ_m). A non-linear regression model as $\log K(\theta) = a + b \log K_a$ with the correlation coefficient(R) ranging from 0.856 to 0.942(significant at P<0.01) were established for the 22 soils grouped into five textural classes. The slope (b) and intercept (a) varied within narrow ranges of -2.504 to -3.65 and -23.4 to -28.73, respectively. For the comparison purpose $K(\theta)$ were also predicted from *RETC* using experimental *SMC* data and van Genuchten and Brooks-Corey models. The reliability of the $K(\theta)$ prediction from K_a based on root mean square deviation (*RMSD*), geometric mean error ratio (GMER), and geometric standard deviation of error ratio (GSDER) criteria became considerable smaller than those predicted from the two models implying that rapid and simple prediction of $K(\theta)$ from K_a is quite promising.

Key Words

Air permeability, Brooks-Corey, hydraulic conductivity, van Genuchten model

Introduction

When water or air passes through a soil at particular water content, water takes liquid filled pores and air takes gas filled pores. Evidently the increase in soil water content raises water permeability expressed as $K(\theta)$ and lowers air permeability (K_a), and thus it is speculated that a close relation or function between the two must exist. Direct measurement of K_a is easier and much rapid than $K(\theta)$ and in contrast to water, establishing air flow through a soil seldom may change or alter the pore geometry, which often occurs during K_s or $K(\theta)$ measurements. There are several studies that have attempted to relate K_s to K_a. Schjoning (1986) presented an exponential relation predicting K_s from K_a at -1 mH_2O in 405 examined soil cores. Blackwell *et al.* (1990) concluded that the volume and depth of samples did not alter the nature of the equation derived between K_s and K_a. Poulsen *et al.* (1999) developed a K_s model based on total and air filled porosity at -1 mH_2O matric potential. Loll and Schjoning (1999) carried out similar study with emphasis on the application of predicted K_s in infiltration modeling. In spite of the cited studies carried out for estimating K_s from K_a, predicting $K(\theta)$ from K_a as far as the authors know has not yet been investigated. The aim of this research was to study about the possibility of fast and reliable prediction of $K(\theta)$ from K_a.

Methods

Twenty-two soil series with nine various textural classes were selected from Karaj, Varamin and Urmia plains in Iran. Bulk and particle densities, soil texture and saturated hydraulic conductivity were measured by routine laboratory methods and using both disturbed and core samples taken from 0-10 cm depth. Volumetric water content of each core sample at 0.25, 0.35, 0.70 mH_2O moisture suctions were determined by water hanging column and at 1, 2, 3, 5 and 10 mH_2O by pressure plate apparatus.
Air permeability (K_a) was determined by the falling head method (Taylor and Ashcroft 1972) in each soil core after its equilibration at various matric potentials (Ψ_m)using hanging columns or pressure plate apparatus. The Eq. [1] was used for the K_a computation.

$$K_a = -2.303 \frac{V\eta\delta_s(\log P_2 - \log P_1)}{AP_a\Delta t} \qquad (1)$$

where V is the chamber volume, η is viscosity, δ_s is soil core length, P_1 and P_2 are the initial and final air pressures in the chamber, A is the area(m^2), P_a is the atmospheric air pressure (kPa), and Δt is the time that air flows through the soil core. With the appropriate dimensions (Taylor and Ashcroft 1972) applied to the variables in Eq. [1], K_a of each core at various Ψ_m (or water contents) were computed in Darcy. 1 Darcy represents the number of m^3 of air with 1 Nsm^{-3} viscosity passing in one second through a unit cross section area of the soil core under the pressure gradient of 1Nm^{-2} per meter. Unsaturated hydraulic conductivity $K(\theta)$ of the soil cores were computed from Eq.[2] by using the experimental data from the pressure plate outflow method (Ghildyal and Tripathi 2001) at 5 or 7 various suctions.

$$K(\theta) = 4\beta Q_0 \rho_w g L^2 / \pi^2 V \Delta P \tag{2}$$

where β is the slope of the regression line plotted as $log\,[Q_0 - Q_t]$ against time (t), Q_0 is total outflow volume(m^3) at the pressure increment (ΔP), Q is the volume(m^3) of water released at various times(t) over the ΔP, ρ_w is the density of water (Mg m^{-3}), g is the acceleration due to gravity (m s^{-2}) and L is the length of the core in the flow direction (m). With applying appropriate dimension to the variables (Q_0, V, ΔP and β) and to the constants (ρ_w, g and L) as described by Ghildyal and Tripathi (2001), $K(\theta)$ was computed in/ms for the average Ψ_m at each successive pairs of pressures P_1 and P_2. Measured $K(\theta)$ were regressed against K_a for each individual soil core, for the soils within each texture class and for the whole 22 examined soils, and the regression equations were developed. The values of $K(\theta)$ at various Ψ_m (or water contents) were also estimated from van Genuchten-Mualem and Brooks-Corey-Mualem models (van Genuchten 1980) using experimental SMC data and RETC software (van Genuchten *et al.* 1991). The following statistical criteria (Wagner *et al.* 2001) were employed for the comparison of the $K(\theta)$ predicted from air permeability and from the two mentioned models.

$$RMSD = \left[\frac{1}{n}\sum_{i=1}^{N}\left(X_{m,i} - X_{p,i}\right)^2\right]^{1/2} \tag{3}$$

$$\varepsilon_i = \frac{X_{p,i}}{X_{m,i}} \tag{4}$$

$$GMER = exp\left(\frac{1}{n}\sum_{i=1}^{n} ln(\varepsilon_i)\right) \tag{5}$$

$$GSDER = exp\left[\frac{1}{n-1}\sum_{i=1}^{n}\left[ln(\varepsilon_i) - ln(GMER)\right]^2\right]^{1/2} \tag{6}$$

$X_{m,i}$ and $X_{p,i}$ refer to the measured and predicted $K(\theta)$ at specified suctions or water contents, respectively, and ε_i denotes their ratio.

Table 1. The regression coefficients (*a* and *b*), and correlation coefficient R between log$K(\theta)$ and log K_a for 22 examined soils.

Soil No.	Soil texture class	Db(kg/m^3)	A	B	R	n
1	Sandy loam^{+}	1460	-30.48	-3.909	0.982**	15
2	Silty loam^{+}	1390	-21.04	-2.121	0.914**	21
3	Loam^{+}	1360	-25.67	-2.953	0.961**	21
4	Loam^{+}	1390	-24.00	-2.632	0.951**	21
5	Silty Clay Loam^{++}	1270	-20.72	-2.102	0.920**	21
6	Silty clay^{++}	1230	-38.96	-5.503	0.967**	21
7	Loam^{+}	1470	-32.72	-4.280	0.921**	21
8	Sand^{+}	1490	-29.92	-4.148	0.880**	21
9	Clay loam^{++}	1550	-25.75	-3.221	0.977**	21
10	Sandy loam^{+}	1520	-28.85	-3.731	0.889**	21
11	Sandy clay loam^{++}	1620	-26.43	-3.072	0.997**	15
12	Sandy clay loam^{++}	1630	-22.86	-2.388	0.985**	15
13	Silty clay^{++}	1510	-22.13	-2.283	0.963**	15
14	Silty clay^{++}	1550	-21.86	-2.252	0.970**	15
15	Sandy clay loam^{++}	1390	-26.80	-3.305	0.961**	15
16	Loam^{+}	1280	-23.34	-2.442	0.924**	15
17	Silty loam^{+}	1440	-25.08	-2.787	0.988**	15
18	Loam^{+}	1540	-19.65	-1.661	0.989**	15
19	Loam^{+}	1480	-28.23	-3.515	0.976**	15
20	Silty loam^{+}	1500	-20.97	-1.961	0.964**	15
21	Silty clay^{++}	1460	-25.07	-2.999	0.972**	15
22	Loam^{+}	1460	-33.63	-5.188	0.732**	11
Mean	-	1454	-26.09	-3.111	0.944	-
C.V	-	7.22	18.57	32.82	-	-

$^{+}$Constant and $^{++}$Falling pressure methods for measuring K_s

Results

The regression and correlation coefficients (*a, b* and *R*) relating log $K(\theta)$ to logK_a for individual soils is depicted by (Table 1). Negative slopes (*b*) demonstrate that with increasing K_a (decrease in water filled pores) $K(\theta)$ diminishes. The logarithmic nature of the regression implies that the rate of decrease in $K(\theta)$ is much more rapid than the rate of increase in K_a. This is expected because of exponential or power dependence of $K(\theta)$ to θ (Hillel 1998). Except for the soils 8, 10 and 22, for the other soils (Table 1) *R* values exceeded 0.92 being significant at 1% probability level which implies that on the average basis about 88% (R^2=0.88) of the variation in $K(\theta)$ in the 0.25 to 10 mH_2O moisture suction range is attributed to the variation in K_a. Large shrinkage in the core samples of soil number 22 particularly at low Ψ_m led to erratic K_a measurements and decreased its *R* to 0.733 even though it is still statistically significant (P< 0.01). The plot of $K(\theta)$ against K_a ($\log K(\theta) = a + b \log K_a$) for the five groups of soils falling in the same textural classes is depicted by Figure 1.

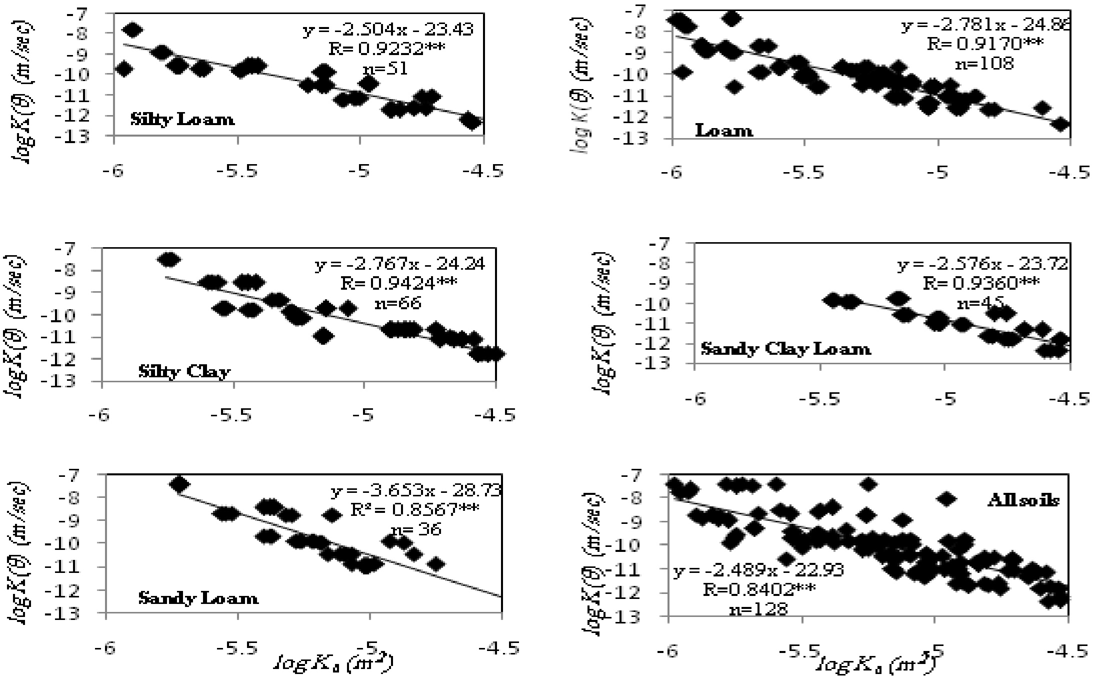

Figure 1. Regression equation and coefficient of correlation (*R*) between water $K(\theta)$ and air (K_a) permeability for the five textural class (ordinate axis is in log scale).

There is no particular trend in the variation of *R* or *a* and *b* of the regression equations with soil textural class. Grouping the examined soils according to their texture, however, improved correlation coefficient (*R*); it ranged from 0.856 to 0.942. The ranges of *a* and *b* values also became narrower. Their variations did not exceed 7.5% and 4%, respectively, implying that the developed regression equations for the texture classes in Figure1 may be reliably applied to other soil falling into the appropriate texture class.

Table 2. Comparison of the prediction reliability of $K(\theta)$ from measured K_a and from van Genucthen and Brooks-Corey models (van Genuchten 1980).

Prediction method	Average of RMSD	$1-\frac{1}{n}\sum_{i=1}^{22} GMERi$	Average of GSDER
van Genuchten-Mualem	1.053	-0.022	1.207
Brooks-Corey-Mualem	0.996	0.200	1.259
Regression method (from K_a)	0.651	0.012	1.060

The averages of three reliability criteria computed for the predicted $K(\theta)$ from K_a using the regression equations in Figure 1 and from van Genuchten and Brooks-Corey models (van Genuchten 1980) is summarized in Table 2.

Average *RMSD* of 0.651 is much lower for the regression method implying that $K(\theta)$ has been predicted with greater reliability than the two methods. The second comparison criterion for the $K(\theta)$prediction is *GMER*. When the value of the expression " $1-\frac{1}{n}\sum_{i=1}^{22} GMER_i$" is equal to zero it corresponds to an exact match between the measured and predicted $K(\theta)$. When greater than zero it implies over-prediction and when less than zero under prediction of $K(\theta)$ relative to the measured values. Table 2 again reveals that $K(\theta)$,s predicted from K_a are more reliable than those predicted by van Genuchten and Brooks-Corey models that currently are employed in many studies. Negative value for "$1-\frac{1}{n}\sum_{i=1}^{22} GMER_i$" implies that van Genuchten model for 22 soils produced slightly under prediction where as Brooks-Corey slightly over prediction of $K(\theta)$. Average of *GSDER*, for 22 soils again is smallest for the $K(\theta)$ predicted from K_a.

Conclusion

For the 22 examined soils results revealed an existence of strong and significant (P<0.01) correlation between K_a and $K(\theta)$ in the range of 0 to -10 mH_2O matric potential. If the relationship for the large number of soils could be generalized, it appears a very effective way for $K(\theta)$ prediction. Reliability of the estimates turned to be better than the well known models such as van Genuchten or Brooks-Corey. Possibility of determining the regression equation coefficients relating $K(\theta)$ to K_a from the easily measured soil attributes needs further investigation.

Acknowledgment

The authors wish to thanks Tabriz University for its support during the course of M.Sc thesis work by Sayed Alireza Rafiee Alavi

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